USN | | | | | | | | | | |

12MMD/MCM/MDE/MEA/MAR/MST11

First Semester M.Tech. Degree Examination, February 2013
Applied Mathematics

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Time: 3 hrs. Max. Marks:100

Note: *Answer any FIVE full questions.*

1 a. Define : i) Inherent error ii) Round off error iii) Truncation error
Round off the numbers 865250 and 37.46235 to four significant figures and find the relative error in each case. **(10 Marks)**

b. A parachutist of mass 68.1 kg jumps out of a stationary hot air balloon. Use i) analytical method and ii) numerical method to compute velocity prior to opening the chute if the drag co-efficient is 12.5 kg/s. (Take a step size of 2 sec for computation). Plot the graph also. **(10 Marks)**

2 a. Derive the formula to compute the root of the equation f(x) = 0 of the False position method. Use it to find out x_5 with four decimals when $x\log_{10} x = 1.2$. **(10 Marks)**

b. Explain with suitable equation, the Newton-Raphson method to find the root of the equation f(x) = 0. Use it to find the root of the equation $3x = \cos x + 1$ near $x_0 = 0.6$. **(10 Marks)**

3 a. State the main steps involved in Bairstow's Lin's method find the quadratic factor as $x^2 + px + q$. Use it to find the quadratic factor when $x^4 + x^3 + 2x^2 + x + 1 = 0$, with the initial values $p_0 = 0.5 = q_0$ **(10 Marks)**

b. If $a_0x^3 + a_1x^2 + a_2x + a_3 = 0$, mention the main steps by squaring three times by Graffe's root squaring method to find the roots. Find the roots of the equation $x^3 - 2x^2 - 5x + 6 = 0$ by squaring three times. **(10 Marks)**

4 a. Derive the expressions for y′ and y″ for Newton's forward and backward interpolation formulas. Find y′ and y″ at x = 3.0 given that **(10 Marks)**

x	3.0	3.2	3.4	3.6	3.8	4.0
y	-14.0	-10.0	-5.3	0.26	6.67	14.0

b. Derive Simpson's 3/8th rule and Weddle's rule starting from general quadrature formula. Use Romberg integration to compute $\int_0^1 \frac{dx}{1+x}$ to three decimal places. Take h as 0.5, 0.25, 0.125. **(10 Marks)**

5 a. Solve $3x + y + 2z = 3$, $2x - 3y - z = -3$, $x - 2y + z = 4$ by Cramer's rule. **(06 Marks)**

b. Solve $2x_1 + 4x_2 + x_3 = 3$, $3x_1 + 2x_2 - 2x_3 = -2$, $x_1 - x_2 + x_3 = 6$ by Gauss elimination method. **(07 Marks)**

c. Apply Cholesky method to solve $3x + 2y + 7z = 4$, $2x + 3y + z = 5$, $3x + 4y + z = 7$. **(07 Marks)**

6 a. State the necessary steps involved in Given's method for tridiagonal matrix. Find the tridiagonal form of the matrix $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 4 & 2 \\ 3 & 2 & 3 \end{bmatrix}$ using Given's method. **(10 Marks)**

b. Use Jacobi's method to find all the eigen values and the eigen vectors of the matrix, $\begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$. **(10 Marks)**

7 a. Define a linear transformation $T : R^2 \to R^2$ by $T(\alpha) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -x_2 \\ x_1 \end{bmatrix}$, find the images under T of $u = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$, $v = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $u + v = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$. **(06 Marks)**

b. The columns of $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, suppose T is a linear transformation from R^2 into R^3 such that $T(e_1) = \begin{bmatrix} 5 \\ -7 \\ 2 \end{bmatrix}$ and $T(e_2) = \begin{bmatrix} -3 \\ 8 \\ 0 \end{bmatrix}$ with no additional information. Find a formula for the image of an arbitrary x in R^2. **(07 Marks)**

c. Prove that the transformation is linear if,
i) $T(u + v) = T(u) + T(v)$, for all u, v.
ii) $T(Cu) = CT(u)$, for all u and C. **(07 Marks)**

8 a. Show that $\{u_1, u_2, u_3\}$ is an orthogonal set where $u_1 = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$, $u_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$, $u_3 = \begin{bmatrix} -1/2 \\ -2 \\ 7/2 \end{bmatrix}$. **(06 Marks)**

b. If $y = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$ and $u = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$, find the orthogonal projections of y onto u. Write y as the sum of two orthogonal vectors, one in span $\{u\}$ and one orthogonal to u. **(07 Marks)**

c. If $W = \text{span}\{x_1, x_2\}$ with $x_1 = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$, $x_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$, construct the orthogonal basis $\{v_1, v_2\}$ for w. **(07 Marks)**

* * * * *